BMI Analysis
Jim Blilie, December 2018

This is my little explanation (or rant) about why Body Mass Index (BMI) is not a valid metric to judge the weight of an individual, based on their height.

I am 6 feet, 5 inches tall. By the BMI metric, I am classified as "Severely Obese" (even after all my weight loss in 2017-2018). I don't think anyone who knows me would consider me to be severely obese (but the insurance companies do, by blindly following the BMI). I don't think any truly severely obese person could keep up with me while hiking uphill or bicycling.

To not be considered overweight, by the BMI I would have to weigh 211 pounds or less. I haven't weighed 211 pounds since I was a skinny, weak 19-year-old. I freely admit that I am overweight; but 211 pounds? You've got to be kidding. By the BMI metric the middle of the range defined as "Normal" is: 183 pounds. I have never weighed 183 pounds since I attained my full adult height at about age 17. By the BMI, someone my height could weigh 156 pounds and be considered of normal weight.

Why do I care? Besides not liking to be viewed as less fit than I am, the BMI has become ubiquitous in doctor's offices and insurance companies and it unfairly penalizes someone of my height. I have been denied life insurance eligibility because of my BMI. This is wrong. BMI should not be used to judge the weight/health of individuals.

Here's why.

First off, the people that introduced the metric in 1972 agreed: It's not a useful predictor for health for individuals, only for comparing populations.

> The modern term "body mass index" (BMI) for the ratio of human body weight to squared height was coined in a paper published in the July 1972 edition of the Journal of Chronic Diseases by Ancel Keys and others. In this paper, Keys argued that what he termed the BMI was "...if not fully satisfactory, at least as good as any other relative weight index as an indicator of relative obesity".[1]
>
> The interest in an index that measures body fat came with observed increasing obesity in prosperous Western societies. <u>Keys explicitly judged BMI as appropriate for population studies and inappropriate for individual evaluation</u>.   [emphasis added]
>
> Nevertheless, due to its simplicity, it has come to be widely used for preliminary diagnoses. Additional metrics, such as waist circumference, can be more useful.[2]

Also from Wikipedia:

> BMI is proportional to the mass and inversely proportional to the square of the height. So, if all body dimensions double, and mass scales naturally with the cube of the height, then BMI doubles instead of remaining the same. This results in taller people having a reported BMI that is uncharacteristically high, compared to their actual body fat levels. In comparison, the Ponderal index is based on the natural scaling of mass with the third power of the height.[3] [emphasis added]

And experts agree:

> "The B.M.I. tables are excellent for identifying obesity and body fat in large populations, but they are far less reliable for determining fatness in individuals," explained Dr. Carl Lavie, a cardiologist at the Ochsner Heart and Vascular Institute in New Orleans.[4] [emphasis added]

In addition, the BMI ignores differences in body build:

> The BMI overestimates roughly 10% for a large (or tall) frame and underestimates roughly 10% for a smaller frame (short stature). In other words, persons with small frames would be carrying more fat than optimal, but their BMI reflects that they are normal. Conversely, large framed (or tall) individuals may be quite healthy, with a fairly low body fat percentage, but be classified as overweight by BMI.[5]

Because it is simple (and commonly used), it has become ubiquitous as the metric for judging the weight of individuals based on their height, despite the fact that the scientists who introduced it specifically warned against this.

As noted in the above quote, for tall people this results in an unrealistically high statistic: The BMI makes you look fatter than you are. Being a person at the 99th percentile in height, this of interest to me.

BMI is the wrong metric

Why is BMI the wrong metric? Because it uses the square of height instead of the cube of height. The formula for BMI (for inches and pounds) is shown in Equation 1.

Equation 1: $$\text{BMI} = \frac{mass}{height^2} \times 703$$

Weight is proportional to volume (assuming relatively uniform density, which is a reasonable assumption for human bodies). And volume is proportional to the cube of the dimension. A simple example of a cylinder is shown in Figure 2. The volume is: Radius X Radius X Length (times the constant, *pi*): inches X inches X inches = in^3. A cylinder is a pretty good estimate for the shape of human body segments.

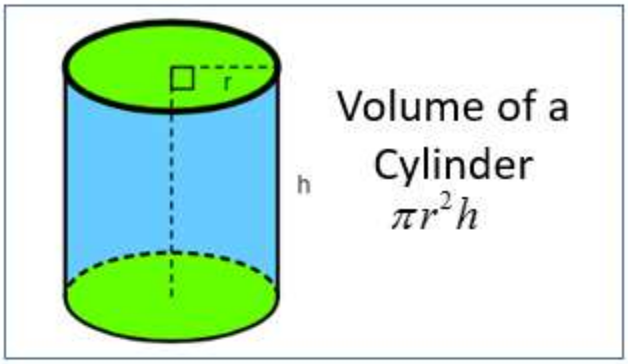

Figure 1: Formula and Diagram for the Volume of a Cylinder

Real Life Example

For a perhaps clearer example than in humans, let's compare two familiar animals for height and weight. They are in different taxonomic families; but both are four-legged herbivores living in the same environment: The African elephant (*Loxodonta africana*) and the Thomson's gazelle (*Eudorcas thomsonii*). They are very different in size: The elephant varies in adult height from 126-157 inches (10.5–13.1 ft.) and the gazelle varies in adult height from 22–32 inches (both from Wikipedia). The ratio of these heights cubed is between 118 and 188 (in contrast, the ratio of the heights squared is between 24 and 33; BMI uses height squared).

The elephant's adult weight varies from 10,362–13,334 lb. and the gazelle's adult weight varies from 44 to 77 pounds. The ratio of weights is between 173 and 236. See Table 1. It's clear that the cubic estimate is much closer to the real weight ratio; although it too underestimates the actual weight ratio (the weight is actually higher than predicted by the cube of the height ratio). The height squared ratio is absurdly low.

This takes the model to its extremes and shows how it doesn't work. Of course it was never meant for elephants; and the human range of height is much narrower. Refer to Figure 3. But it shows clearly that the square of the height is the wrong factor. The 99^{th} percentile human (77 inches) is 1.45 times taller than the 1% human (53 inches)[6], while the elephant is approximately seven times taller than the Thomson's gazelle.

Also look at Figure 2: Notice how different the body proportions are between the two animals. The elephant's body sections (torso, limbs) and much thicker than those of the gazelle. This is driven by the fact that weight (loading) increases with the cube of length (or height) and so the load-bearing segments must by relatively thicker (stronger) for the taller/longer animal to carry those loads.[1]

Table 1: Relative Height and Weight Ratios for African Elephant and Thomson's Gazelle

Animal	Height Range (inch)	Weight Range (pound)	Ratio of Height Cubed	Ratio of Height Squared	Ratio of Weights
Elephant	126-157	10,362–13,334	118 - 188	24 - 33	173 - 236
Gazelle	22-32	44-77			

Figure 2: Photos of an African Elephant and a Thomson's Gazelle

[1] The limbs won't be increased in dimension by a cubic ratio because the strength is proportional to area (axial loading) which is proportional to the square of the dimension; or, for bending, strength is proportional to a more complex formula where the dimension is raised to the third power (in net effect).

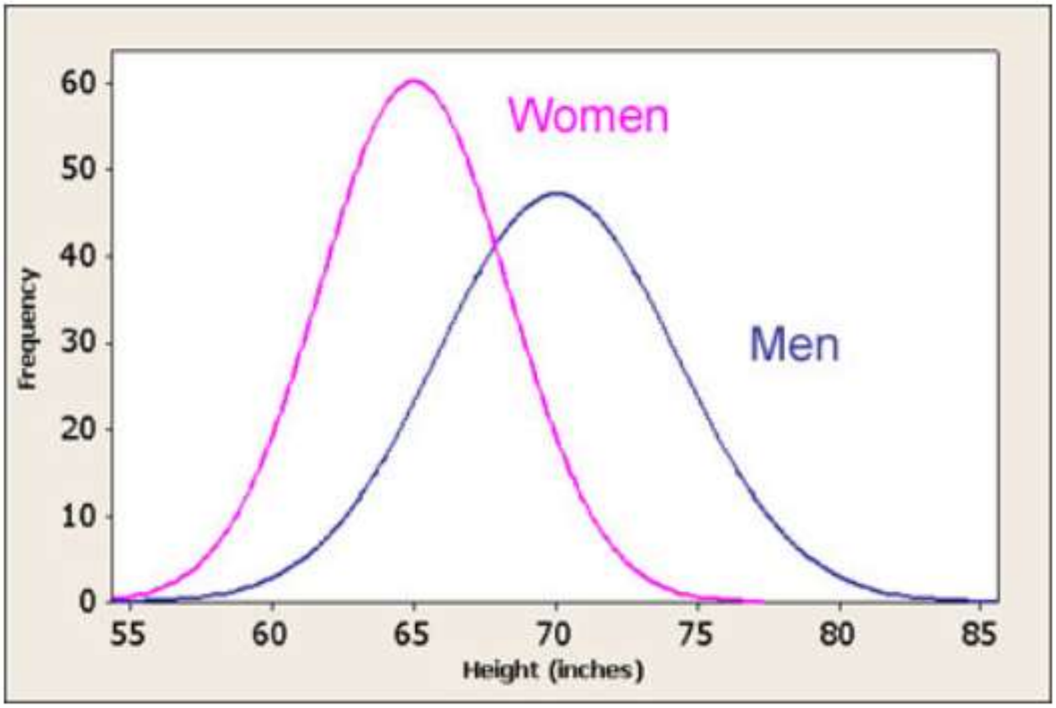

Figure 3: Distributions of Human Height

Dimension #	Dimension	Measurements			
		95th Percentile Male		5 th Percentile Female	
		Metric	Imperial	Metric	Imperial
	Weight	102 kgs	225 #	49 kgs	108 #
1	Standing Height	186.5 cms	73.4 ins	151.5 cms	59.6 ins
5	Hip Height	100.0 cms	39.4 ins	74.0 cms	29.1 ins
8	Erect Sitting Height	97.0 cms	38.2 ins	79.5 cms	31.3 ins
10	Sitting Shoulder Height	64.5 cms	25.4 ins	50.5 cms	19.9 ins
17	Sitting Shoulder Width	50.5 cms	19.9 ins	37.5 cms	14.8 ins
19	Hip Width	40.5 cms	15.9 ins	31.0 cms	12.2 ins
25	Shoulder Grip Length	71.5 cms	28.1 ins	55.5 cms	21.9 ins
30	Foot Length - bare	28.5 cms	11.2 ins	22.0 cms	8.7 ins
31	Foot Width - bare	11.0 cms	4.3 ins	8.5 cms	3.3 ins

Figure 4: 5th Percentile and 95th Percentile Human Dimensions[7]

How Does BMI Look?

Let's look at plots of the BMI categories based on height. Figure 5 shows the lower bounds of the various categories for the BMI.

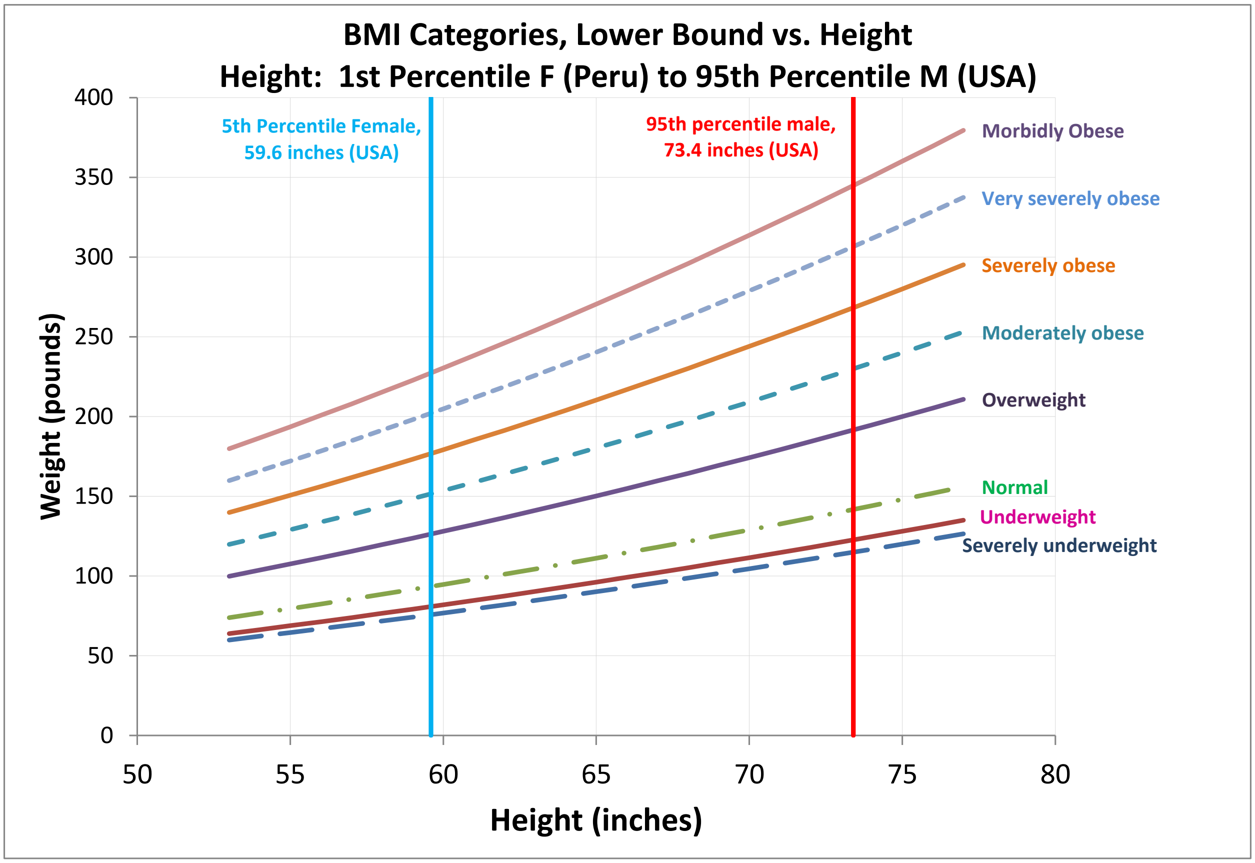

Figure 5: Lower Bounds (Weight) of BMI Categories Plotted Against Height

Next let's put some reference lines on this plot of BMI. Figure 6 shows the BMI categories again; but with the addition of the plots of the volumes of cylinders of three different aspect ratios (heavy black dashed lines). As noted above, a cylinder is a pretty good model for human body segments, refer to Figure 1. The aspect ratios[2] model most human body parts well. The scale on the right (cylinder volume) was adjusted so that the plots cross the BMI category lower bounds (Morbidly obese, Overweight, Severely Underweight) at the 1% height.

This is one way to show how the BMI will underestimate weight at the taller end of the scale. The fact is that human weight will be directly proportional to volume (within reasonable tolerances) and volume is proportional to the cube of length.

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[2] Aspect ratio in this case is the length of the cylinder divided by its diameter. This is held constant and the volume is driven by the length.

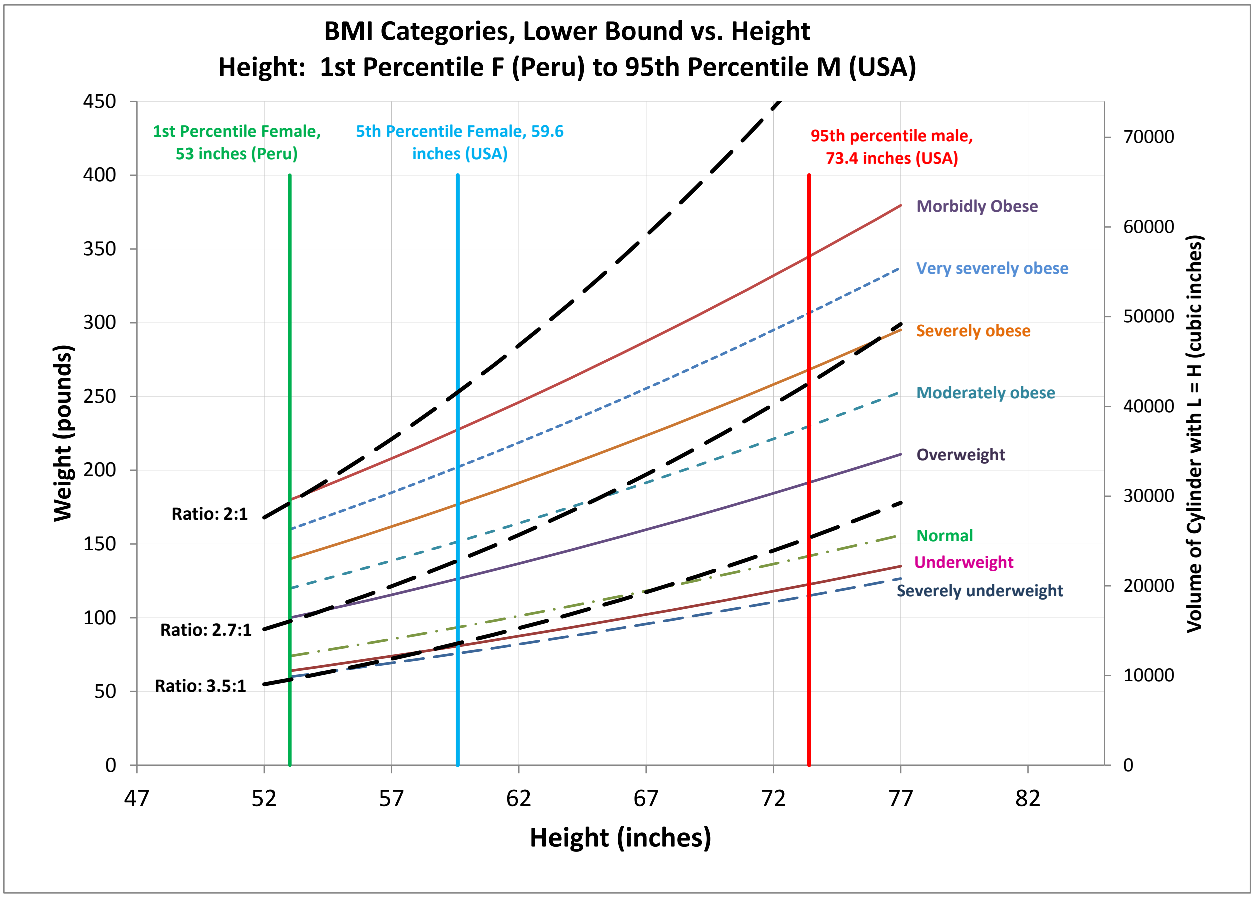

Figure 6: BMI Categories with Cylinder Volumes for Reference.

I assume that the BMI is based on a small height (if you based it on the high (tall) end of the scale, then short people would be overestimated by it for weight). Remember that the numbers for the category bounds were arbitrarily chosen. The person proposing the BMI would pick a height and weight for a given category and then shorter or taller than that exact height, error would begin to accumulate. The further you get from that chosen base height, the greater the error, in both directions. I assume it was based on a small height because the biggest errors are at the tall end of the range.

A Better BMI

I would propose to modify the BMI to a cubic BMI, based on the cube of the height. Given what we know about height, volume, and mass, this is bound to be more accurate. Refer to Equation 1.

Equation 2: $$\text{BMI} = \frac{mass}{height^3} \times constant$$

Just like in the BMI, there's a constant to make things match up. It's 703 for the BMI (in inches and pounds) so that the inches and pounds end up at the same BMI number as that for meters and kilograms.

In order to match up with the BMI, one has to choose a constant. One chooses the constant to make the BMI numbers match at a certain location. Figure 7 shows a cubic BMI with the matching at the lower bound of the Normal Category and at height = 66 inches. (Some of the BMI categories were removed for clarity; the cylinder volumes were retained.) Obviously, the BMI curves now track to the cylinder curves, since they are both cubic. For this matching point, the constant (Equation 2) turns out to be: 46,300. The lower bound for overweight for someone my height (77 inches) turns out to be 247 pounds. In my estimation, this is a decent match, but maybe a bit high.

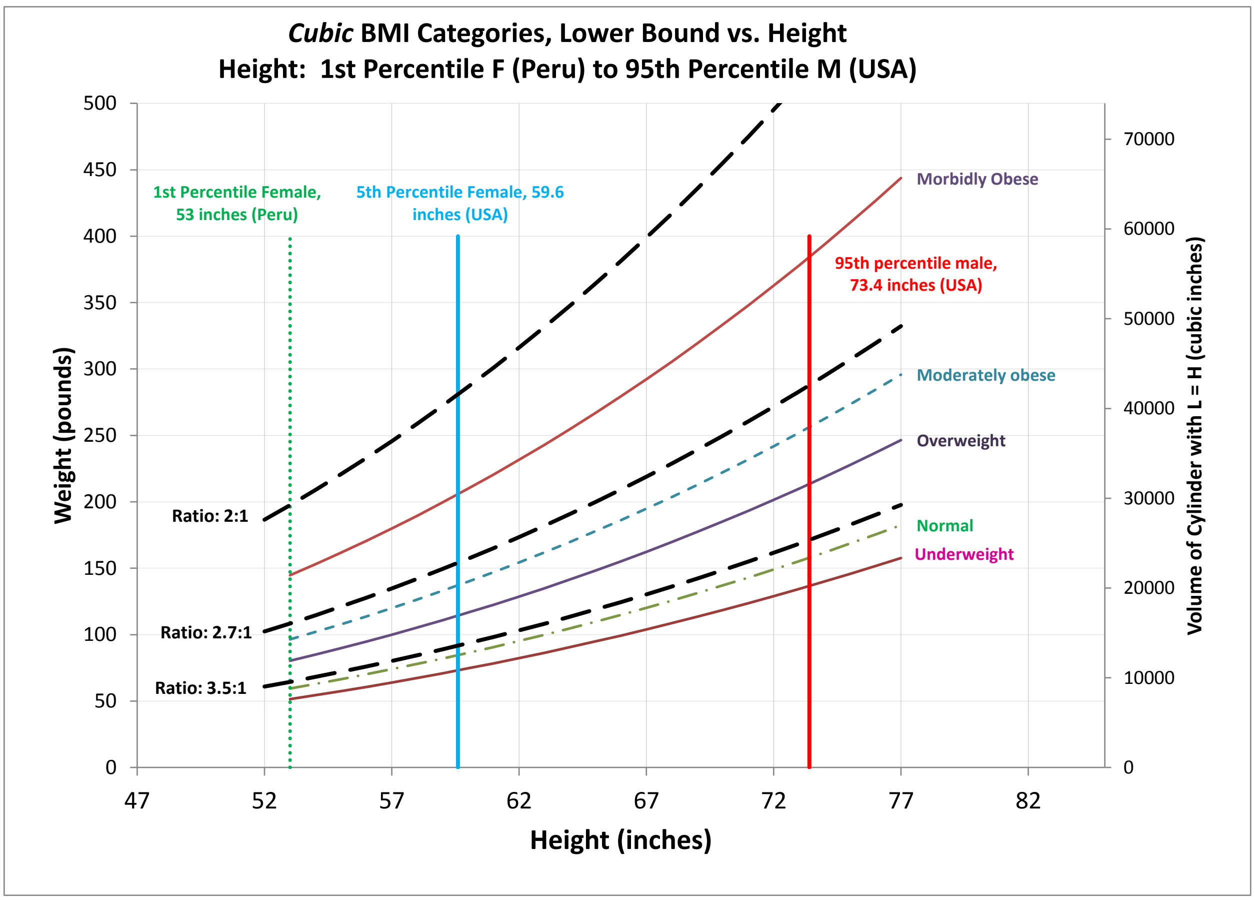

Figure 7: Cubic BMI with Matching at Lower Bound of Normal and 66-inch Height

Figure 8 shows a cubic BMI with the matching at the center of the Normal Category and at height = 60 inches. For this matching point, the constant (Equation 2) turns out to be: 42,400. The lower bound for overweight for someone my height (77 inches) turns out to be 269 pounds. This is too high. Since we based the model on a short person (five feet tall) more error accumulated out at a height of 77 inches.

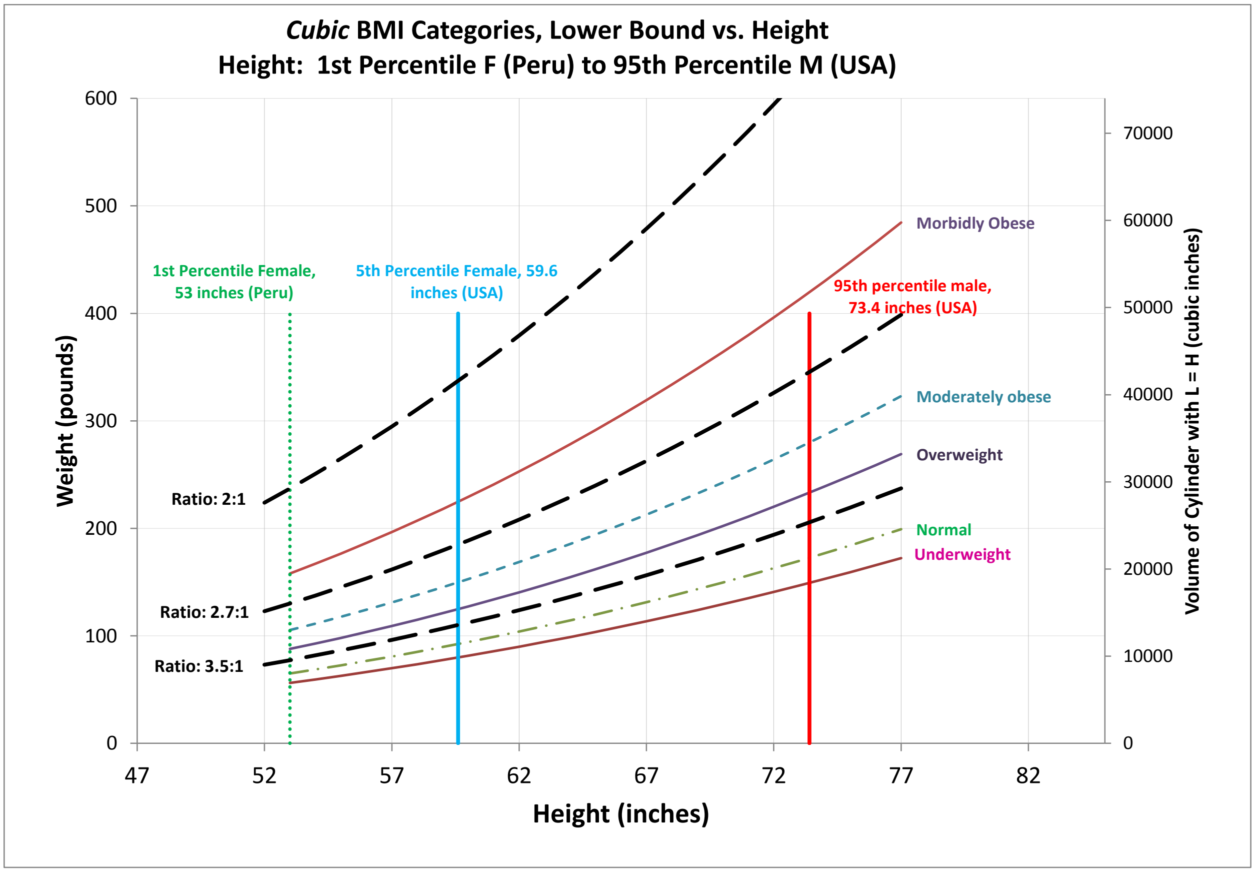

Figure 8: Cubic BMI with Matching at Center of Normal and 60-inch Height

Figure 9 shows a cubic BMI with matching at the center of the Normal Category and height of 69 inches (average male height in the USA). This means it matches best at the middle of normal and average male height. For this matching point, the constant (Equation 2) turns out to be: 48,700. The lower bound for overweight for someone my height (77 inches) turns out to be 234 pounds.

In my estimation, this is a good model, at least for someone my height. Again, as you move away from the height where the model is based, error accumulates. If you base the model on the middle of the Normal Category and at the average height (for men) then it will have the least overall error, most likely (again, for men).

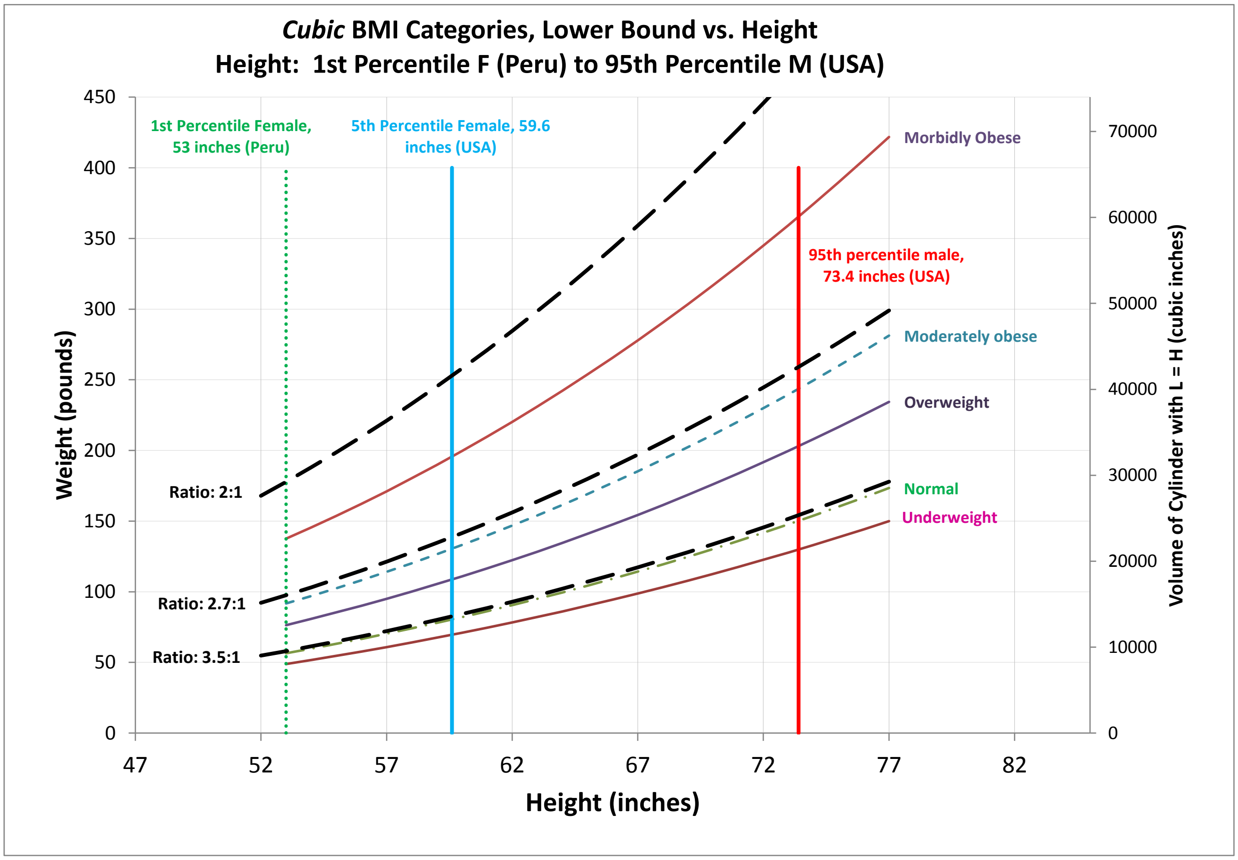

Figure 9: Cubic BMI with Matching at Center of Normal and 69-inch Height

Conclusion:

- No model is accurate at all heights, for both genders, and for all geographic locations.
- The current BMI, based on the square of the height, is a bad model. As was recommended by the people who introduced the metric: It should not be used to judge the weight of individuals.
- The cubic model of BMI is much better, since it conforms with physics and real bodies. But, as we have seen, where you base the model makes a difference.
- Don't judge someone's weight by the current BMI model!

[1] Keys A, Fidanza F, Karvonen MJ, Kimura N, Taylor HL (July 1972). "Indices of relative weight and obesity". Journal of Chronic Diseases. **25** (6): 329–43; republished as: Ancel Keys, Flaminio Fidanza, Martti J Karvonen, Noburu Kimura and Henry L Taylor (2014). "Indices of relative weight and obesity", International Journal of Epidemiology, 2014, 655–665

[2] Source: https://en.wikipedia.org/wiki/Body_mass_index

[3] Source: https://en.wikipedia.org/wiki/Body_mass_index

[4] Source: https://www.nytimes.com/2010/08/31/health/31brod.html

[5] https://www.medicalnewstoday.com/articles/265215.php

[6] Source: https://tall.life/height-percentile-calculator-age-country/

[7] Source: SAE rules: https://www.fsaeonline.com/content/FSAE%20Rules95th_2016.pdf These values are used for design.